Oscillating modulation to B-mode polarization from varying propagating speed of primordial gravitational waves

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Abstract

In low-energy effective string theory and modified gravity theories, the propagating speed c_T of primordial gravitational waves may deviate from unity. We find that the step-like variation of c_T during slow-roll inflation may result in an oscillating modulation to the B-mode polarization spectrum, which can hardly be imitated by adjusting other cosmological parameters, and the intensity of the modulation is determined by the dynamics of c_T. Thus provided that the foreground contribution is under control, high-precision CMB polarization observations will be able to put tight constraint on the variation of c_T, and so the corresponding theories.

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I. INTRODUCTION

Inflation, as the paradigm of the early universe, has not only solved a lot of fine-tuning problems of the big bang theory, but also predicted the primordial scalar and tensor perturbations. The primordial tensor perturbations, i.e. primordial gravitational waves (GWs) [1][2][3], have arisen great attentions after the BICEP2 collaboration's announcement of the detection of B-mode signal in the CMB (around $l \sim 80$) [4], which was interpreted by them as the imprint of the primordial GWs, though this result is doubtful due to the foregrounds of polarized dust emissions [5][6], see also [7].

The detection of primordial GWs would verify general relativity (GR) and strengthen our confidence in inflation and quantum gravity[8], and also put more constrains on inflation models and modified gravity at the same time. Besides the CMB experiments which mainly aimed at detecting low frequencies ($10^{-17} - 10^{-15}$ Hz) GWs, many experiments relate to higher frequencies based on other methods, such as pulsar timing array ($10^{-9} - 10^{-8}$ Hz), laser interferometer detectors ($10^{-4} - 10^{4}$ Hz), will be carried out in the coming decades. However, since the amplitude of the GWs would stay constant after they are stretched outside the horizon, and decrease with the expansion of the universe after they reenter the horizon, the primordial GWs with longer wavelength provide the most of opportunities for the detection [9]. Therefore, the CMB observations, especially the CMB B-mode detections, are still the most promising experiments to detect the primordial GWs if they actually exist.

Einstein's GR is the most accepted theory of gravity. However, it might be required to modify when dealing with the inflation in the primordial universe and the accelerated expansion of the current universe. During the matter and radiation dominated era, modified gravity has several effects on the CMB spectra, such as the lensing contribution to B-modes [10] and the variation of propagating speed c_T of primordial GWs [11][12], we are especially interested in the latter in this paper, see e.g.[13] for the case with the scalar perturbation. In GR, graviton is massless and propagates along the null geodesics, so the propagating speed c_T of GWs is naturally set to be unity, i.e. the speed of light. But in modified gravity, e.g., the low-energy effective string theory with high-order corrections [14][15][16][17][18], and also modified Gauss-Bonnet gravity [19], and generalized Galileon (Horndeski theory [20]) [21][22][23][24], and beyond Horndeski theories [25],[26], and the effective theory of fluids at next-to-leading order in derivatives (e.g.[27]), c_T might deviate from unity. Because the

value of c_T determines the time of horizon crossing of GWs, during the recombination the change of c_T can result in a shift of the peak position of the primordial B-modes, see [11][12], for example.

In this paper, we focus on the effect of the variation of c_T during slow-roll inflation on the CMB B-mode polarization, and show how it offers a distinct way to test the modified gravity theories. We find that the step-like variation of c_T may result in an oscillating modulation to the B-mode polarization spectrum, which can hardly be imitated by adjusting other cosmological parameters. The intensity of the modulation is determined by the ratio of c_T before and after the variation, which depends on the dynamics of theoretical models. This oscillating modulation is so rich in feature that it may easily be discriminated from the variation of other parameters or other features. Thus provided that the foreground contribution is under control, high-precision CMB polarization observations will be able to put tight constraint on the variation of c_T, and so the corresponding theories.

II. OSCILLATING SPECTRUM OF PRIMORDIAL GWS

We begin with the action for the GWs mode h_{ij}, e.g.[17][23]

$$S_{(2)} = \int d\tau d^3x \, \frac{a^2 Q_T}{8} \left[h_{ij}'^2 - c_T^2 (\vec{\nabla} h_{ij})^2 \right], \tag{1}$$

where h_{ij} obeys $\partial_i h_{ij} = 0$ and $h_{ii} = 0$, Q_T is regarded as effective Planck scale $M^2_{P,eff}(\tau)$, c_T is the propagating speed of primordial GWs, and the prime is the derivative with respect to the conformal time τ, $d\tau = dt/a$. During slow-roll inflation, the slow-roll parameter $\epsilon = -\dot{H}/H^2 \ll 1$, as well as

$$\epsilon_Q = \frac{\dot{Q}_T}{HQ_T} \ll 1, \quad s = \frac{\dot{c}_T}{Hc_T} \ll 1 \tag{2}$$

are required, e.g., see [28] for a recent study.

Here, we will mainly focus on the effect of varying c_T, i.e., the condition $s \ll 1$ might be broke at some point, on primordial GWs spectrum. Noting that the effects of varying sound speed of primordial scalar perturbations on the scalar power spectrum have been investigated in [29][30]. We do not get entangled with the details of (1) and the evolution of background, and will assume that the background is the slow-roll inflation, which is not affected by the variation of c_T, and set Q_T constant and $M_P^2 = 1$. We will discuss a possibility of such a

case in Appendix A. In addition, there may be a mass term [31][32] in (1), which might also be time-dependent [33], but we will not involve it.

We can expand h_{ij} into Fourier series as $h_{ij}(\tau, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^3} e^{-i\mathbf{k}\cdot\mathbf{x}} \hat{h}_{ij}(\tau, \mathbf{k})$, where

$$\hat{h}_{ij}(\tau, \mathbf{k}) = \sum_{\lambda=+,\times} \left[h_\lambda(\tau, k) a_\lambda(\mathbf{k}) + h^*_\lambda(\tau, -k) a^\dagger_\lambda(-\mathbf{k}) \right] \epsilon^{(\lambda)}_{ij}(\mathbf{k}), \tag{3}$$

where the polarization tensors $\epsilon^{(\lambda)}_{ij}(\mathbf{k})$ are defined by $k_j \epsilon^{(\lambda)}_{ij}(\mathbf{k}) = 0$, $\epsilon^{(\lambda)}_{ii}(\mathbf{k}) = 0$, which satisfy $\epsilon^{(\lambda)}_{ij}(\mathbf{k}) \epsilon^{*(\lambda')}_{ij}(\mathbf{k}) = \delta_{\lambda\lambda'}$, $\epsilon^{*(\lambda)}_{ij}(\mathbf{k}) = \epsilon^{(\lambda)}_{ij}(-\mathbf{k})$, the commutation relation for the annihilation and creation operators $a_\lambda(\mathbf{k})$ and $a^\dagger_\lambda(\mathbf{k}')$ is $[a_\lambda(\mathbf{k}), a^\dagger_{\lambda'}(\mathbf{k}')] = \delta_{\lambda\lambda'} \delta^{(3)}(\mathbf{k} - \mathbf{k}')$. We define $h_\lambda(\tau, k) = u_k(\tau)/z_T$ and $z_T = a\sqrt{Q_T}/2$. Thus we get the equation of motion for u_k as

$$u''_k + \left(c_T^2 k^2 - \frac{z''_T}{z_T} \right) u_k = 0. \tag{4}$$

To phenomenologically investigate the effect on primordial GWs spectrum induced by varying c_T, we assume that the variation of c_T can be described by a step-like function

$$c_T = \begin{cases} c_{T1} & (\tau < \tau_0) \\ c_{T2} & (\tau > \tau_0) \end{cases}, \tag{5}$$

where $\tau_0 < 0$ is the transition time.

We take the background evolution to be the slow-roll inflation. Of course, it is also provided that the sudden change of c_T won't affect the background evolution. Then, we have $z''_T/z_T \equiv a''/a \approx (2 + 3\epsilon)/\tau^2$, and the equation of motion (4) becomes

$$u''_k + \left(c_T^2 k^2 - \frac{\nu^2 - 1/4}{\tau^2} \right) u_k = 0, \tag{6}$$

where $\nu = \sqrt{\frac{9}{4} + 3\epsilon} \approx \frac{3}{2} + \epsilon$. The solution to Eq.(6) is familiar, we can write it as

$$\begin{aligned} u_{k1} &= \sqrt{-c_{T1}k\tau} \left[C_{1,1} H^{(1)}_\nu(-c_{T1}k\tau) + C_{1,2} H^{(2)}_\nu(-c_{T1}k\tau) \right], \quad \tau < \tau_0, \\ u_{k2} &= \sqrt{-c_{T2}k\tau} \left[C_{2,1} H^{(1)}_\nu(-c_{T2}k\tau) + C_{2,2} H^{(2)}_\nu(-c_{T2}k\tau) \right], \quad \tau > \tau_0, \end{aligned} \tag{7}$$

where $H^{(1)}_\nu$ and $H^{(2)}_\nu$ are the first and second kind Hankel functions of ν-th order, respectively. These coefficients C are functions of the comoving wave number k, but constants with respect to conformal time τ. $C_{1,1}$ and $C_{1,2}$ are determined by the initial condition.

Here, we set the initial condition as the standard Bunch-Davies(BD) vacuum. Therefore, when $c_{T1}k \gg \frac{a''}{a}$, which corresponds to perturbations deep inside the horizon,

$$u_k \sim \frac{1}{\sqrt{2c_{T1}k}} e^{-ic_{T1}k\tau}. \tag{8}$$

When $c_{T1}k \gg \frac{a''}{a}$, u_{k1} in Eq.(7) should approximate to Eq.(8). Allow for the Hankel function $H_v^{(1)}(\xi) = \sqrt{\frac{2}{\pi\xi}} e^{i(\xi - \frac{v\pi}{2} - \frac{\pi}{4})}$ and $H_v^{(2)}(\xi) = \sqrt{\frac{2}{\pi\xi}} e^{-i(\xi - \frac{v\pi}{2} - \frac{\pi}{4})}$ when $|\xi| \to \infty$, we get

$$C_{1,1} = \frac{\sqrt{\pi}}{2\sqrt{c_{T1}k}}, \quad C_{1,2} = 0. \tag{9}$$

The coefficients $C_{2,1}$ and $C_{2,2}$ are determined by requiring u_k and u_k' to be continuous at $\tau = \tau_0$, i.e. the matching condition. Then we obtain

$$\begin{aligned} C_{2,1} = \frac{i\pi^{\frac{3}{2}}\tau_0 k^{\frac{1}{2}}}{16\sqrt{c_{T2}}} \Big[& \; c_{T1} \left(H^{(1)}_{-1+\nu}(-c_{T1}k\tau_0) - H^{(2)}_{1+\nu}(-c_{T1}k\tau_0) \right) H^{(2)}_{\nu}(-c_{T2}k\tau_0) \\ & + \; c_{T2} \left(-H^{(2)}_{-1+\nu}(-c_{T2}k\tau_0) - H^{(2)}_{1+\nu}(-c_{T2}k\tau_0) \right) H^{(1)}_{\nu}(-c_{T1}k\tau_0) \Big], \\ C_{2,2} = \frac{i\pi^{\frac{3}{2}}\tau_0 k^{\frac{1}{2}}}{16\sqrt{c_{T2}}} \Big[& \; c_{T1} \left(-H^{(1)}_{-1+\nu}(-c_{T1}k\tau_0) + H^{(1)}_{1+\nu}(-c_{T1}k\tau_0) \right) H^{(1)}_{\nu}(-c_{T2}k\tau_0) \\ & + \; c_{T2} \left(H^{(1)}_{-1+\nu}(-c_{T2}k\tau_0) - H^{(1)}_{1+\nu}(-c_{T2}k\tau_0) \right) H^{(1)}_{\nu}(-c_{T1}k\tau_0) \Big]. \end{aligned} \tag{10}$$

The spectrum of primordial GWs is defined by $P_T = (k^3/2\pi^2)\langle 0|\hat{h}_{ij}(\tau, -\mathbf{k})\hat{h}_{ij}(\tau, \mathbf{k})|0\rangle$ with $\tau \to 0$, which is only a function of comoving wave number k. After neglecting the slow-roll parameter, from Eq.(7) with $\nu = 3/2$, we have

$$|u_{k2}| = \frac{\sqrt{2}}{-c_{T2}k\tau\sqrt{\pi}} \left| C_{2,1} - C_{2,2} \right|. \tag{11}$$

Therefore, we obtain the power spectrum of primordial GWs as

$$P_T \; = \; \frac{k^3}{2\pi^2} \sum_{\lambda=+,\times} |h_\lambda(\tau, k)|^2 = P_T^{inf} \frac{4k}{Q_T \pi c_{T2}^2} \left| C_{2,1} - C_{2,2} \right|^2, \tag{12}$$

where

$$P_T^{inf} = 2H_{inf}^2/\pi^2 \tag{13}$$

is that of standard slow-roll inflation without modified gravity, i.e. $Q_T = 1$ and $c_{T1} = c_{T_2} = 1$, and H_{inf} is the Hubble parameter during inflation, which sets the scale of inflation.

The effect of varying c_T is encoded in $C_{2,1}$ and $C_{2,2}$. We set $x = c_{T2}/c_{T1}$ and defined a new function

$$f(k, k_0, x) = \frac{4c_{T1}k}{\pi x^2} \left| C_{2,1} - C_{2,2} \right|^2, \tag{14}$$

where $k_0 = -1/(c_{T1}x\tau_0)$. Then, the GWs spectrum (12) may be rewritten as

$$P_T = P_T^{inf} \cdot \frac{f(k, k_0, x)}{c_{T1}^3 Q_T}, \tag{15}$$

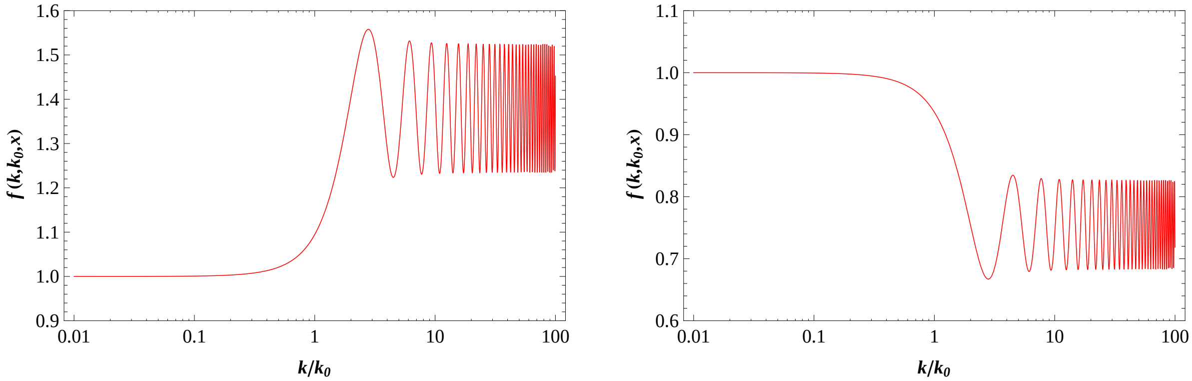

FIG. 1: The function $f(k, k_0, x)$, where $x = c_{T2}/c_{T1}$. We have set $x = 0.9$ on the left panel and $x = 1.1$ on the right panel, respectively.

where $f(k, k_0, x)$ is obtained as

$$f(k, k_0, x) = \frac{1}{x^2}\sin^2\left(\frac{k}{k_0}\right) + \frac{1}{x^4}\left[\cos(\frac{k}{k_0}) - (1-x^2)\frac{k_0}{k}\sin(\frac{k}{k_0})\right]^2 . \tag{16}$$

We plot $f(k, k_0, x)$ with respect to k/k_0 in Fig.1, in which we set $x = 0.9$ on the left panel and $x = 1.1$ on the right panel, respectively. Here, the transition time $\tau_0 = -1/(c_{T1}xk_0)$ sets a character scale $1/k_0$. When $k \ll k_0$, i.e. the perturbation mode has longer wavelength than $1/k_0$, we have $f(k, k_0, x) \approx 1$, and $P_T = P_T^{inf}/(c_{T1}^3 Q_T)$ is scale invariant, which is the result of slow-roll inflation with almost constant c_T and Q_T [23][34]. When $k \gg k_0$, we have

$$f(k, k_0, x) \approx \frac{1}{x^2}\left[1 + (\frac{1}{x^2} - 1)\cos^2(\frac{k}{k_0})\right] , \tag{17}$$

thus $f(k, k_0, x)$ oscillates between $1/x^2$ and $1/x^4$, and P_T oscillates correspondingly, just as we can see from Fig.1.

In the case of varying sound speed c_S of primordial scalar perturbations, the sudden change of c_S may lead to the oscillating modulation to the primordial scalar spectrum, as well as the CMB TT-mode spectrum, just as found in [29]. In addition, the oscillation in the primordial scalar spectrum can also be attributed to some other effects, such as inflaton potential with a small oscillation [35][36], a sudden change in inflaton potential or its derivative, e.g., [37][38][39][40]. Thus the oscillation in the primordial scalar spectrum may be implemented without modified gravity, as has been mentioned.

However, the oscillation in the primordial GWs spectrum can only be attributed to the modified gravity. When the gravity is GR, P_T equals to P_T^{inf}, which is given in Eq.(13). The

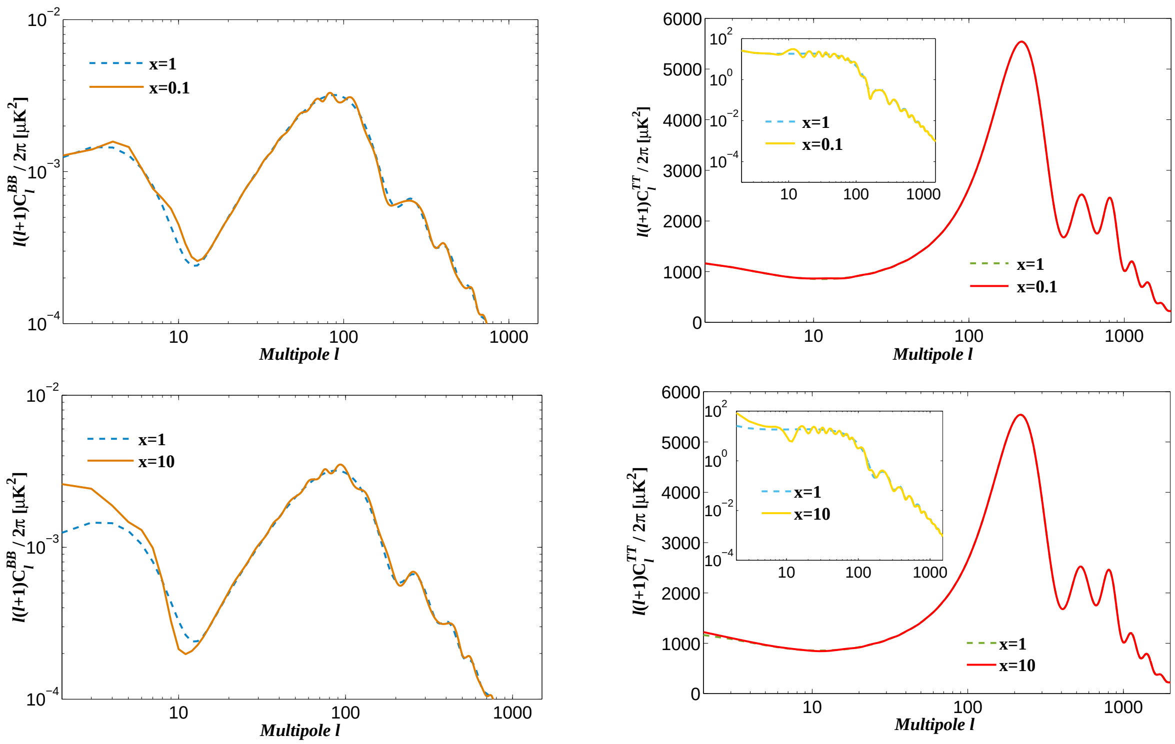

FIG. 2: Theoretical CMB BB and TT-mode power spectra for our oscillating GWs spectrum (15)(brown line in the left panel and red solid line in the right panel) and the power-law GWs spectrum for reference(blue dashed line in the left panel and green dashed line in the right panel). The insets of the right panels are the TT-mode spectra for our oscillating GWs spectrum (the yellow solid lines) and the power-law GWs spectrum (the blue dashed lines) for reference. We set $r = 0.05$ and $k_0 = 1/30000\,\mathrm{Mpc}^{-1}$.

oscillation of P_T^{inf} certainly requires H_{inf} is oscillating, which is impossible, unless the null energy condition is violated periodically. Though the particle production may also modify the GWs spectrum [41][42][43], it only leads to a bump-like contribution, which is entirely different from the behavior of oscillation.

III. CMB B-MODE POLARIZATION SPECTRUM

The primordial GWs is imprint in CMB as the B-mode polarization. Thus the oscillation in the primordial GWs spectrum will inevitably affect the B-mode polarization spectrum.

To see such effects, we plot the CMB BB and TT-mode correlations in Fig.2, in which

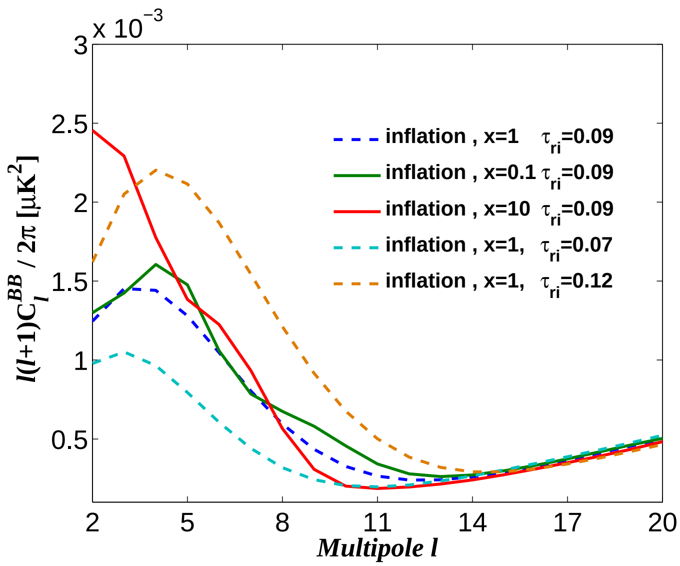

FIG. 3: BB-mode spectra at low multipoles for our oscillating GWs spectrum (15) with different x(solid lines) and the power-law GWs spectrum with different τ_{ri}(dashed lines).

P_T^{inf} in (15) is parameterized as

$$P_T^{inf} = r A_{\mathcal{R}}^{inf} \left(\frac{k}{k_*}\right)^{n_T^{inf}} . \tag{18}$$

Here, we assume that the scalar spectrum is hardly affected by the modified gravity, which will be clarified in Appendix A. Thus the scalar perturbation spectrum is set as $P_{\mathcal{R}}^{inf} = A_{\mathcal{R}}^{inf} (k/k_*)^{n_{\mathcal{R}}^{inf}-1}$, in which $A_{\mathcal{R}}^{inf}$ is the amplitude of the scalar perturbations. In addition, we also assume that after inflation the propagating speed c_T is unity, so that the spectrum of B-mode polarization is not affected by relevant evolution at late time, or see [11],[12].

In the left panels of Fig.2, we see some obvious enhancements or suppressions to the reionization bump in the BB-mode spectrum, which depend on the oscillating effect on corresponding scales. The height of the reionization bump can be estimated roughly as [44],

$$C_{T,l\sim 2}^{BB} \approx \frac{1}{100} \left(1 - e^{-\tau_{ri}}\right)^2 C_{T,l\sim 2}^{TT}, \tag{19}$$

where $C_{T,l}^{TT}$ stands for the TT-mode spectrum from the primordial GWs without the reionization and τ_{ri} is the optical depth to the beginning of reionization. The periodic enhancements and suppressions of the reionization bump are a reflection of the oscillations of primordial GWs spectrum on large scales. In addition, we can also see some obvious oscillations around the recombination peak at $l \sim 80$.

In the right panels of Fig.2, we see that the TT-mode spectrum is hardly affected by the oscillating primordial GWs power spectrum, since the contribution of GWs to TT-mode spectrum is negligible, compared with the scalar perturbations. The case of EE-mode polarization spectrum is actually also similar. Thus the main effect of varying speed of primordial GWs is on the B-mode polarization, which makes the B-mode polarization spectrum show its obvious enhancements or suppressions to the reionization bump and oscillations around the recombination peak.

In Ref.[45], the authors have pointed out that it is possible to set to one the propagating speed of GWs by a proper redefinition of the metric. They got the gravitational waves spectrum in their Eq.(12) "same" as the standard one in the form. However, since they have made a redefinition of the time coordinate and the scale factor in Eq.(9), the variation of their $\tilde{H}$ with respect to $\tilde{t}$ after redefinition is not the same as the variation of H with respect to t. Therefore, the oscillating feature induced by a step-like c_T is encoded in $\tilde{H}$, the result in both frame should be the same.

It might also be a concern whether such a B-mode polarization spectrum can be imitated by adjusting other cosmological parameters or not. In Eq.(19), the optical depth τ_{ri} is relevant with the height of the reionization bump. We show the BB-mode spectrum with different τ_{ri} in Fig.3. We see that the change of τ_{ri} can only alter the overall amplitude of BB-mode spectrum at low multipoles, but hardly create the oscillation. In addition, in inflationary models with pre-inflation era, the reionization bump could also be suppressed (e.g.[46],[47],[48],[49]) or enhanced (e.g.[50]). However, similarly, these models also only alter the overall amplitude of the BB-mode spectrum, without oscillation, at low multipoles. These results indicate that although the BB-mode spectrum may be modified by other ways, the oscillating modulation leaded by varying the speed of primordial GWs is difficult to be imitated. Thus the measure of B-mode polarization spectrum provides an appropriate way for testing the corresponding gravity physics in the primordial universe.

IV. DISCUSSION

In low-energy effective string theory and modified gravity theories, the propagating speed c_T of primordial GWs may deviate from unity. We calculated the spectrum of primordial GWs, assuming that c_T has a step-like variation and the background of slow-roll inflation

is not affected by it. We found the spectrum of primordial GWs acquires an oscillating modulation, which makes the B-mode polarization spectrum show its obvious enhancements or suppressions to the reionization bump and oscillations around the recombination peak. The intensity of the modulation is determined by c_{T2}/c_{T1}. The frequency of the modulation is determined by $k_0 = -1/(c_{T2}\tau_0)$. Both depend on the dynamics of theoretical models.

The oscillating behavior of the B-mode polarization can only be attributed to the effect of modified gravity, since it can hardly be imitated by adjusting other cosmological parameters. The oscillating behavior is so rich in feature that it may be easily discriminated from the variation of other parameters or other features, thus the upcoming CMB experiments, such as CMBPol, B-Pol, will be able to put a tight constraint on the propagating speed of primordial GWs, and so the corresponding theories, provided that the foreground contribution is under control. In a certain sense, our paper again highlights the significance of B-mode polarization measures in exploring the fundamental physics of primordial universe.

Here, we only postulate a simple step-like variation of c_T, which, however, might be far complicated in some modified gravity models, as well as accompanied by the variation of Q_T. The effect could be non-trivial in more general cases, which is under study. But in a certain sense, a smooth change of c_T will induce oscillations too, see e.g.[51] for the case of scalar perturbations, so we expect that the case of GWs is similar. When we focus on the B-mode polarization spectrum, the assumption we adopted is that after inflation the propagating speed c_T is unity, which may also be relaxed. Moreover, it may well be possible to produce oscillatory features beyond the standard slow-roll background. The varying c_T and Q_T will also affect the non-Gaussianities of primordial perturbations. The relevant issues are open.

Acknowledgments

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Appendix A:

In this Appendix, we will argue how to realize the change of c_T and the changeless of Q_T in (1) in low-energy effective string theory and modified gravity theories.

In low-energy effective action of string theory, the simplest extension of the lowest-order

action is e.g.[17]

$$\mathcal{L}_{correction} \sim -\frac{\alpha' \lambda \xi(\varphi)}{2} \left(c_1 R_{GB}^2 + c_2 G^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \right) \tag{20}$$

where $G^{\mu\nu} = R^{\mu\nu} - g^{\mu\nu} R/2$, and $R_{GB}^2 = R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$ is the Gauss-Bonnet term, α' is the inverse string tension, λ is a parameter allowing for different species of string theories, c_1 and c_2 are coefficients. We have neglected the terms with $\Box\varphi$ and $(\partial_\mu\varphi\partial^\mu\varphi)^2$, since both do not contribute to GWs.

The introducing of (20) will affect not only the GWs, but also the adiabatic scalar perturbations, of course, it is interesting to check its effect on the latter. However, for our purpose, we will regard φ as the spectator field, so that the effects of (20) on the background and the scalar perturbations are negligible.

The action for GWs is (1), and [17]

$$\begin{aligned} Q_T &= 1 - \frac{\alpha' \lambda}{2} \left(8 c_1 \dot{\xi} H_{inf} - c_2 \xi \dot{\varphi}^2 \right), \\ c_T^2 &= \frac{1}{Q_T} \left[1 - \frac{\alpha' \lambda}{2} \left(c_2 \xi \dot{\varphi}^2 + 8 c_1 \ddot{\xi} \right) \right], \end{aligned} \tag{21}$$

see also Ref.[52] for that with $c_1 = 0$. When $8 c_1 \dot{\xi} H_{inf} \equiv c_2 \xi \dot{\varphi}^2$ is imposed, $Q_T = 1$. Then, we have

$$c_T^2 = 1 - 4 c_1 \alpha' \lambda (\dot{\xi} H_{inf} + \ddot{\xi}). \tag{22}$$

The step-like variation of c_T^2 requires that $\dot{\xi} H_{inf} + \ddot{\xi}$ has the step at $\tau = \tau_0$.

As an example, we will give a numerical result of the variation of c_T^2 and φ with respect to cosmological time t in the case of $x = c_{T2}/c_{T1} = 10$. According to Refs.[14][17], we adopt

$$\xi(\varphi) = -e^{-\varphi}, \;\; c_1 = -1, \;\; \alpha' = 1, \;\; \text{and}\, \lambda = -\frac{1}{8}\, (\text{for Heterotic string}). \tag{23}$$

We take the expression of $\varphi(t)$ as

$$\varphi(t) = t - \ln \left[b_1 e^t + b_2 A - A e^t - \frac{A e^{t - t^2}}{\sqrt{\pi}} + A t e^t + A e^{\frac{1}{4}} \mathrm{erf}(\frac{1}{2} - t) - A e^t (t-1) \mathrm{erf}(t) \right], \tag{24}$$

where $A = -1 + 1/x$, and $\mathrm{erf}(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-t^2} dt$ is the Gauss error function, b_1 and b_2 are some constants. We plot φ and c_T^2 with respect to cosmological time t in Fig.4.

Of course, the variation of c_T^2 could be more complicated than a step-like evolution, which would be harder to deal with. However, as it may, $Q_T > 0$ and $c_T^2 > 0$ should be required to avoid ghost and gradient instabilities.

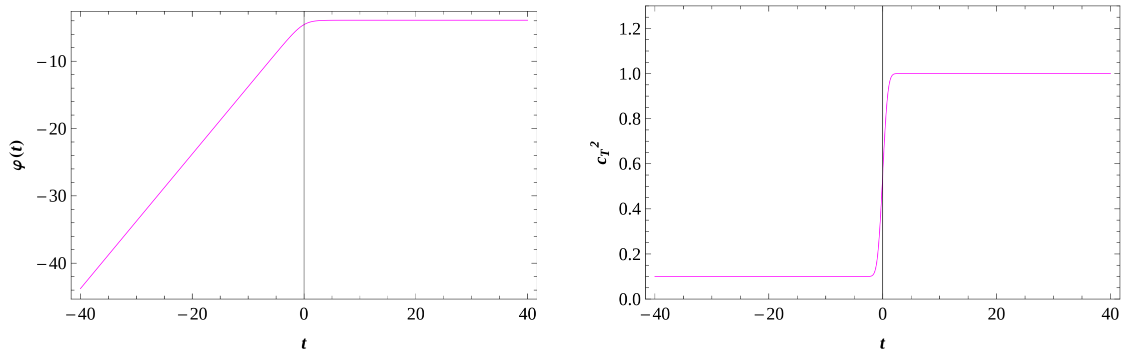

FIG. 4: The variation of φ and c_T^2 with respect to cosmological time t in the case of $x = c_{T2}/c_{T1} = 10$, i.e., $A = -0.9$, where $t = 0$ corresponds to τ_0. We have set $b_1 = 50$, $b_2 = -50$.

The action (20) is actually equivalent to a subclass of the Horndeski theory, see Ref.[23], and so the analysis is also similar for the Horndeski theory.

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[1] L. P. Grishchuk, Sov. Phys. JETP **40**, 409 (1975).

[2] A. A. Starobinsky, JETP Lett. **30**, 682 (1979).

[3] V. A. Rubakov, M. V. Sazhin, and A. V. Veryaskin, Phys. Lett. **B115**, 189 (1982).

[4] P. A. R. Ade *et al.* [BICEP2 Collaboration], Phys. Rev. Lett. **112**, 241101 (2014) [arXiv:1403.3985 [astro-ph.CO]].

[5] R. Adam *et al.* [Planck Collaboration], arXiv:1409.5738 [astro-ph.CO].

[6] P. A. R. Ade *et al.* [BICEP2 and Planck Collaborations], [arXiv:1502.00612 [astro-ph.CO]].

[7] M. J. Mortonson and U. Seljak, arXiv:1405.5857 [astro-ph.CO].

[8] A. Ashoorioon, P. S. Bhupal Dev and A. Mazumdar, Mod. Phys. Lett. A **29**, no. 30, 1450163 (2014) [arXiv:1211.4678 [hep-th]].

[9] L. P. Grishchuk, In *Ciufolini, I. (ed.), Matzner, R.A. (ed.): General relativity and John Archibald Wheeler* 151-199 [arXiv:0707.3319 [gr-qc]].

[10] A. Lewis and A. Challinor, Phys. Rept. **429**, 1 (2006) [astro-ph/0601594].

[11] L. Amendola, G. Ballesteros and V. Pettorino, Phys. Rev. D **90**, 043009 (2014) [arXiv:1405.7004 [astro-ph.CO]].

[12] M. Raveri, C. Baccigalupi, A. Silvestri and S. Y. Zhou, arXiv:1405.7974 [astro-ph.CO].

[13] Y. S. Piao, Phys. Rev. D **75**, 063517 (2007) [gr-qc/0609071]; Phys. Rev. D **79**, 067301 (2009)

[arXiv:0807.3226 [gr-qc]].

[14] R. R. Metsaev and A. A. Tseytlin, Nucl. Phys. **B293**, 385 (1987).

[15] I. Antoniadis, J. Rizos and K. Tamvakis, Nucl. Phys. B **415**, 497 (1994) [hep-th/9305025].

[16] C. Cartier, E. J. Copeland and R. Madden, JHEP **0001**, 035 (2000) [hep-th/9910169].

[17] C. Cartier, J. c. Hwang and E. J. Copeland, Phys. Rev. D **64**, 103504 (2001) [astro-ph/0106197].

[18] Y. S. Piao, S. Tsujikawa and X. m. Zhang, Class. Quant. Grav. **21**, 4455 (2004) [hep-th/0312139].

[19] S. Nojiri and S. D. Odintsov, Phys. Lett. B **631**, 1 (2005) [hep-th/0508049]; K. Bamba, A. N. Makarenko, A. N. Myagky and S. D. Odintsov, Phys. Lett. B **732**, 349 (2014) [arXiv:1403.3242 [hep-th]].

[20] G. W. Horndeski, Int. J. Theor. Phys. **10**, 363 (1974).

[21] L. Amendola, Phys. Lett. B **301**, 175 (1993) [gr-qc/9302010].

[22] C. Deffayet, X. Gao, D. A. Steer and G. Zahariade, Phys. Rev. D **84**, 064039 (2011) [arXiv:1103.3260 [hep-th]].

[23] T. Kobayashi, M. Yamaguchi and J. Yokoyama, Prog. Theor. Phys. **126**, 511 (2011) [arXiv:1105.5723 [hep-th]].

[24] Z. -G. Liu, J. Zhang and Y. -S. Piao, Phys. Rev. D **84**, 063508 (2011) [arXiv:1105.5713].

[25] J. Gleyzes, D. Langlois, F. Piazza and F. Vernizzi, arXiv:1404.6495 [hep-th].

[26] X. Gao, Phys. Rev. D **90**, 081501 (2014) [arXiv:1406.0822 [gr-qc]]; X. Gao, Phys. Rev. D **90**, 104033 (2014) [arXiv:1409.6708 [gr-qc]].

[27] G. Ballesteros, arXiv:1410.2793 [hep-th].

[28] A. De Felice and S. Tsujikawa, arXiv:1411.0736 [hep-th].

[29] M. Nakashima, R. Saito, Y. i. Takamizu and J. Yokoyama, Prog. Theor. Phys. **125**, 1035 (2011) [arXiv:1009.4394 [astro-ph.CO]].

[30] H. Firouzjahi and M. H. Namjoo, Phys. Rev. D **90**, 063525 (2014) [arXiv:1404.2589 [astro-ph.CO]].

[31] S. Dubovsky, R. Flauger, A. Starobinsky and I. Tkachev, Phys. Rev. D **81**, 023523 (2010) [arXiv:0907.1658 [astro-ph.CO]].

[32] A. E. Gumrukcuoglu, S. Kuroyanagi, C. Lin, S. Mukohyama and N. Tanahashi, Class. Quant. Grav. **29**, 235026 (2012) [arXiv:1208.5975 [hep-th]].

[33] Q. G. Huang, Y. S. Piao and S. Y. Zhou, Phys. Rev. D **86**, 124014 (2012) [arXiv:1206.5678 [hep-th]].

[34] A. De Felice and S. Tsujikawa, Phys. Rev. D **84**, 083504 (2011) [arXiv:1107.3917 [gr-qc]].

[35] X. Wang, B. Feng, M. Li, X. L. Chen and X. Zhang, Int. J. Mod. Phys. D **14**, 1347 (2005) [astro-ph/0209242].

[36] R. Flauger, L. McAllister, E. Pajer, A. Westphal and G. Xu, JCAP **1006**, 009 (2010) [arXiv:0907.2916 [hep-th]].

[37] A. A. Starobinsky, JETP Lett. **55**, 489 (1992) [Pisma Zh. Eksp. Teor. Fiz. **55**, 477 (1992)].

[38] J. A. Adams, B. Cresswell and R. Easther, Phys. Rev. D **64**, 123514 (2001) [astro-ph/0102236].

[39] M. Joy, V. Sahni and A. A. Starobinsky, Phys. Rev. D **77**, 023514 (2008) [arXiv:0711.1585 [astro-ph]]; M. Joy, A. Shafieloo, V. Sahni and A. A. Starobinsky, JCAP **0906**, 028 (2009) [arXiv:0807.3334 [astro-ph]].

[40] J. Liu and Y. S. Piao, Phys. Lett. B **705**, 1 (2011) [arXiv:1106.5608 [hep-th]].

[41] J. L. Cook, and L. Sorbo, Phys. Rev. D **85**, 023534 (2012) [arXiv:1109.0022 [astro-ph.CO]].

[42] L. Senatore, E. Silverstein and M. Zaldarriaga, JCAP **1408**, 016 (2014) [arXiv:1109.0542 [hep-th]].

[43] S. Mukohyama, R. Namba, M. Peloso and G. Shiu, JCAP **1408**, 036 (2014) [arXiv:1405.0346 [astro-ph.CO]].

[44] S. Saito, K. Ichiki and A. Taruya, JCAP **0709**, 002 (2007) [arXiv:0705.3701 [astro-ph]].

[45] P. Creminelli, J. Gleyzes, J. Nore?a and F. Vernizzi, Phys. Rev. Lett. **113**, no. 23, 231301 (2014) [arXiv:1407.8439 [astro-ph.CO]].

[46] Y. -S. Piao, B. Feng and X. -m. Zhang, Phys. Rev. D **69**, 103520 (2004) [hep-th/0310206];

[47] Z. G. Liu, Z. K. Guo and Y. S. Piao, Phys. Rev. D **88**, 063539 (2013) [arXiv:1304.6527]; Y. T. Wang and Y. S. Piao, Phys. Lett. B **741**, 55 (2015) [arXiv:1409.7153 [gr-qc]].

[48] Z. G. Liu, Z. K. Guo and Y. S. Piao, Eur. Phys. J. C **74**, 3006 (2014) [arXiv:1311.1599 [astro-ph.CO]].

[49] Y. Cai, Y. T. Wang and Y. S. Piao, arXiv:1501.01730 [astro-ph.CO].

[50] R. K. Jain, P. Chingangbam, L. Sriramkumar and T. Souradeep, Phys. Rev. D 82, 023509 (2010) [arXiv:0904.2518 [astro-ph.CO]].

[51] A. Achucarro, V. Atal, B. Hu, P. Ortiz and J. Torrado, Phys. Rev. D **90**, no. 2, 023511 (2014) [arXiv:1404.7522 [astro-ph.CO]].

[52] K. Feng, T. Qiu and Y. S. Piao, Phys. Lett. B **729**, 99 (2014) [arXiv:1307.7864 [hep-th]].